



FORWARD MODEL ANALYSIS IN DIFFUSE OPTICAL TOMOGRAPHY USING RTE AND FEM

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ABSTRACT

Diffuse optical tomography (DOT) is an effective non-invasive medical technique used to capture the images of interior part of the body by handling light in the near-infrared (NIR) region (700 nm to 1200 nm). In DOT, the optical properties of the human tissue such as absorption and scattering coefficients are reconstructed. The objective of the forward model in DOT is to predict the photon flux density at a tissue region. The radiative transfer equation (RTE) and diffusion equation solved by finite element methods (FEM) are used in a forward model to handle complex geometries of the phantom. The main advantages of these methods, compared to other photon transport models, lies in the arrangement of rate and the capability to handle complex geometries as well as it also helps in generating stiffness and power visualization. The above two algorithms predicted the values of optical flux, absorption coefficient and scattering coefficient for normal and cancer affected people. The diffusion equation solution using FEM provides better results compared to the RTE.

Index Terms—Diffuse optical tomography; Near infra-red (NIR); Diffusion equation; Finite element method.

I.INTRODUCTION

Diffuse optical tomography (DOT) [1, 9] is a method of imaging technique using near-infrared (NIR) region for bio-medical applications that has potential to provide the three-dimensional images of the optical parameters such as absorption coefficient and the scattering coefficient [12, 16]. The light transport of photons through a biological tissue is established during diffusion equation. This type of photons is scattered or absorbed in turbid

media. Since the high level of scattering in a most biological tissue medium image reconstruction in DOT is suffering from ill-posed nonlinear problem [7, 19]. It has so many applications in a medical imaging. The biomedical applications of DOT include early stage detection of breast cancer, measurement of brain function and others in bio-measurement fields.

The forward model frequently utilized the light transport models it contains stochastic models such as Monte-Carlo [8] simulation method or deterministic models such as RTE or diffusion

equation (DE). In the previous techniques, photon transport in a biological tissue can be equivalently modeled in numbers with Monte-Carlo simulation [5, 13] method or analytically by RTE. It is referred to as the Boltzmann transport equation (BTE), which is the term of the balance of energy within a boundary volume element of the scattering medium. The analytical results to the RTE [4, 18] for a very easy method but for more practical media; with very complex multiple scattering effects and the numerical methods are required. It has some disadvantages such as difficult to solve without introducing approximation and it is computationally very expensive. So the DOT uses the diffusion equation (DE), which is describing the light transport problem in a biological tissue. The solution to the diffusion equations for photon transports are more computationally efficient and can be analyzed deeply.

The numerical method contains finite difference method (FDM), finite element method (FEM) [1], finite volume method (FVM) and boundary element methods (BEM). The forward model is to predict the optical flux density at the detectors given in a geometric model with optical parameters as source-detector location and functionality. The forward model problem was developed by using the finite element method (FEM) for solving the DE [15] to the RTE. FEM is a numerical method [11] for calculating the approximation solutions to the boundary value problem and to generate the optical properties and optical flux for a given distributions of an absorption and diffusion coefficient. The robin boundary conditions are incorporated at the boundary values. It has the true representation of complex geometry, simple representation of the total solution, generates the spatially resolved images and develops the low-resolution [17,24] of the functional image with high-resolution complementary structures. In the inverse model, the solution of the forward model problem is used for calculating the jacobian matrix and the simultaneous equation is solved using a conjugate gradient function.

Mohammad Ali Ansari et al [20] has provided a volume fraction of blood tissue by light absorption using boundary element method (BEM) with a suitable procession and accuracy. Tissue depth is the main parameter in diffuse optical tomography which BEM has a good ability to reconstruct. However, in wide tissue layers its accuracy decrease due to the small value of data received from reflectance. Young Sik Jun and Woon Sik Baek [21] has developed the light transport in a highly scattered tissue medium. The biological tissue is modeled by using the diffusion equation that is Boltzmann transport equation (BTE). However, it has a large amount of variables for reflection and to obtain the result of BTE is very difficult. Gao and Zhao [22] analyzed the numerical solver for radiative transport equation. The mathematical calculation of RTE is challenging due to its high dimension. Bo Bi and Bo Han [18] analyzed the forward model design of DOT using radiative transfer equation (RTE), which provides accurate and efficient results within biological tissue medium and models the inverse problem using regularization method. In this method, we can get the most accurate results from inner measurements. In an execution time, we cannot obtain the inner measurement data frequently. Hence, the RTE cannot reconstruct inclusions with complicated internal structure accurately with a small amount of boundary angular-averaged measurements. Hence, we have evaluated the finite element method numerically. It is the most flexible one in terms of dealing with complex geometry and difficult boundary conditions. Table I shows the quantities and dimensions of the diffusion equation.

The review of the paper is as follows in section II, we had discussed about the forward modeling. The forward model was described by the diffusion equation (DE) governing boundary value problems; the solution was obtained by radiative transfer equation (RTE) and finite element method (FEM). In section III, results and discussion based on the forward model were analyzed for normal and cancer affected people. In section IV, we discuss in the conclusion that the diffusion equation modeled by FEM performance was better compared to RTE.

II. FORWARD MODEL

The forward model of DOT is to find out the outgoing current on the detectors when the incident impulse and the absorption coefficients and scattering coefficients are known. It describes photon propagation in tissue [14]. In an experimental method of getting potential measurements is as follows. First, a set of laser source [10] devices and detectors on the boundary of the object guess distribute the optical properties absorption and scattering coefficient. The incident impulse launched from the laser source and the resulting measurement from all the detectors was recorded.

TABLE I. DESCRIPTION OF QUANTITIES

S. No	Quantity	Description	Dimension
1.	\bar{f}	Average cosine of (direction independent) phase function	None
2.	$k(r)$	Diffusion coefficient	(m ² s ⁻¹)
3.	\hat{n}	normal unit vector	(mm)
4.	$s_0(r, t)$	isotropic component of the source	(m ⁻³)
5.	P	radial distance	(mm)
6.	$\mu_a(r)$	absorption coefficient at position \mathbf{r}	(m ⁻¹)
7.	$\mu_s(r)$	scattering coefficient at position \mathbf{r}	(m ⁻¹)
8.	$\mu_s'(r)$	reduced scattering coefficient	(m ⁻³)
9.	$\phi(r, t)$	photon density at point \mathbf{r} , at time t	(m ⁻³)
10.	c	speed of light in a medium	(m)

A. Radiative transfer equation

The radiative transfer equation (RTE) [18, 1] describing the photon propagation in tissue medium. The RTE is solved by analytically in a biological tissue; the RTE is commonly accepted model for light propagation in a biological tissue. The time domain in an RTE is given by,

$$\left(\frac{1}{c} \frac{\partial L}{\partial t} + \hat{s} \cdot \nabla L(r, t, \hat{s}) + (\mu_s + \mu_a) L(r, t, \hat{s}) = \mu_s \int_{4\pi} L(r, t, \hat{s}') f(\hat{s}, \hat{s}') d^2 \hat{s}' + q(r, t, \hat{s}) \right) \quad (1)$$

Where, the radiance is denoted by $L(r, t, \hat{s})$, $q(r, t, \hat{s})$ is the source of inside and the scattering phase function is denoted by $f(\hat{s}, \hat{s}')$. The energy transfer per unit time is denoted by L and the photons in a solid angle is $d^2 \hat{s}'$ during an elemental area da is specified by its outgoing normal unit \hat{n} at position \mathbf{r} is set by,

$$L(r, t, \hat{s}) \hat{s} \cdot \hat{n} da d^2 \hat{s}' \quad (2)$$

The exitance Γ with a unit area perpendicular to \hat{n} is achieved by the above integrating equation. Over all angle is obtained by,

$$\Gamma(r, t) = \int_{4\pi} L(r, t, \hat{s}) \hat{s} \cdot \hat{n} d^2 \hat{s}' \quad (3)$$

In RTE, both spatial location and the angular direction of the discretization are implemented and the equations are derived by analytically or stochastically.

B. Diffusion equation

The forward model frequently handles the light transport model such as a diffusion equation (DE). In DOT, the diffusion equation in a frequency domain [4,23] can be obtained by,

$$\left(-\nabla \cdot k(r) \nabla + \left(\mu_a(r) + \frac{j\omega}{c} \right) \right) \phi(r, \omega) = q_0(r, \omega) \quad (4)$$

Where $\omega = 2\pi f$, $k(r)$ is a diffusion coefficient, $\phi(r)$ is a photon flux density at the position \mathbf{r} , c is the

speed of light in a medium and $\mu_a(r)$ is the absorption coefficient. The diffusion coefficient $k(r)$ is given by,

$$k(r) = \left[\frac{1}{3(\mu_a(r) + \mu'_s(r))} \right] \quad (5)$$

μ_a and μ'_s illustrate the absorption and the reduced scattering coefficient ($\mu_a \ll \mu'_s$) [13] respectively. The reduced scattering coefficient is described by $\mu'_s = \mu_s(1-g)$ with μ_s represents the scattering coefficient and the g is an anisotropy factor, with the typical range of 0.7 to 0.9. The photon input is from a source of constant intensity located at $r = r_0$. The output optical flux on the boundary is given by,

$$\Gamma(\xi, t) = -k \frac{\partial}{\partial n} \phi(r, t) \Big|_{\partial\Omega} = -k \hat{n} \cdot \nabla \phi(r, t) \Big|_{\partial\Omega} \quad (6)$$

Where, the normal unit vector to the surface $\partial\Omega$ is denoted by \hat{n} . Type III robin boundary condition [12] is used for diffusion equation, as it models refractive index mismatch at the boundary of the soft tissue. The robin boundary condition is solved using FEM and $\phi(r)$ is obtained.

C. Solution of DE using Finite element method

The light diffusion equation in a frequency domain is numerically calculated by using the finite element method (FEM) [3, 6]. In the FEM, we have applied the final technique in the form well-known as the galerkin formulation method. The Galerkin formulation method [2, 4] proceeds solution of the optical flux ϕ ,

$$\{\nabla \cdot k(r) \nabla - \mu_a(r)c - \frac{\partial}{\partial t}\} \phi(r, t) = -q_0(r, t) d\Omega \quad (7)$$

Where, $s_0(r, t)$ is the source term. If it is also a solution of,

$$\int_{\Omega} \psi(r) \left\{ \nabla \cdot k(r) \nabla - \mu_a(r)c - \frac{\partial}{\partial t} \right\} \phi(r, t) d\Omega = - \int_{\Omega} \psi(r) s_0(r, t) d\Omega \quad (8)$$

Where, $\psi(r)$ is a function from a suitable test space satisfying the same boundary conditions as ϕ . Integration by the parts,

$$\begin{aligned} \int_{\Omega} \psi(r) \nabla k(r) \cdot \nabla \phi(r, t) + \mu_a(r)c \psi(r) \phi(r, t) + \\ \psi(r) \frac{\partial}{\partial t} \phi(r, t) d\Omega = \int_{\Omega} \psi(r) s_0(r, t) d\Omega + \\ \int_{\partial\Omega} \psi(r) \Gamma(r, t) d(\partial\Omega) \end{aligned} \quad (9)$$

For this problem to be solvable, ψ and all first derivatives must be integral over Ω . The solution ϕ to the diffusion equation (4) is approximated by a continuous function,

$$\phi^h(r, t) = \sum_{i=1}^D \phi_i(t) \psi_i(r) v^h \quad (10)$$

such that,

$$\begin{aligned} \int_{\Omega} \psi(r) \nabla k(r) \cdot \nabla \phi(r, t) + \mu_a(r)c \psi(r) \phi(r, t) + \\ \psi(r) \frac{\partial}{\partial t} \phi(r, t) d\Omega = \int_{\Omega} \psi(r) s_0(r, t) d\Omega + \\ \int_{\partial\Omega} \psi(r) \Gamma(r, t) d(\partial\Omega), \\ \forall \psi_{ij} = 1 \rightarrow D \end{aligned} \quad (11)$$

This can be expressed in matrix notation as,

$$[K(k) + C(\mu_a c)] \phi + M \frac{\partial}{\partial t} = Q + \beta \quad (12)$$

Where,

$$\begin{aligned}
 K_{ij} &= \int_{\Omega} k(r) \nabla \psi_i(r) \cdot \nabla \psi_j(r) d\Omega \\
 Q_j(t) &= \int_{\Omega} \psi_i(r) s_0(r, t) d\Omega \\
 \beta_j &= \int_{\partial\Omega} \psi_i(r) \Gamma(r, t) d(\partial\Omega) \\
 C_{ij} &= \int_{\Omega} \mu_a(r) c \psi_i(r) \cdot \nabla \psi_j(r) d\Omega \\
 M_{ij} &= \int_{\Omega} \psi_i(r) \cdot \nabla \psi_j(r) d\Omega \\
 \phi &= [\phi_1(t), \phi_2(t), \dots, \phi_D(t)]^T
 \end{aligned} \tag{13}$$

We assume the boundary condition $\psi_i = 0$ on $\partial\Omega$. The $D \times D$ matrices K, C , and B are a light while, they will have non-zero entries only when N_j and N_i are vertices of the same element.

III. RESULTS AND DISCUSSION

The radiative transfer equation (RTE) and the finite element method (FEM) solution to the forward model design, we consider a two-dimensional circular image of human tissue has 20mm radius with six sources and detectors placed at the boundary of the head. In a simulation method, we execute the programs on a 3.0 GHz window 7 PC with 8 GB RAM in MATLAB 2013a. The optical properties of the forward model were set as 0.01 cm^{-1} absorption and 1.0 cm^{-1} scattering coefficients for normal people.

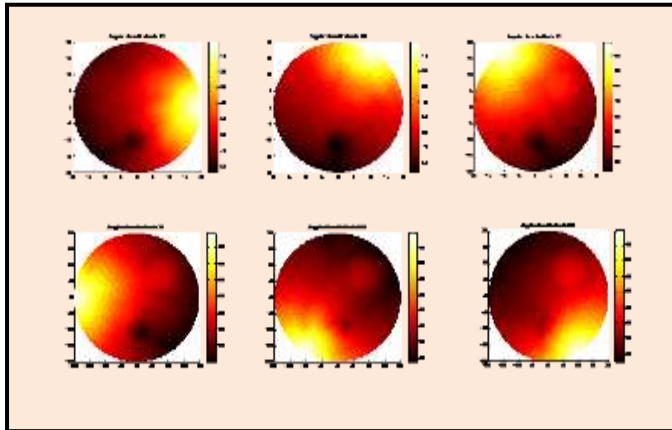


Fig 1. Angular discretization of the 2-D images for RTE

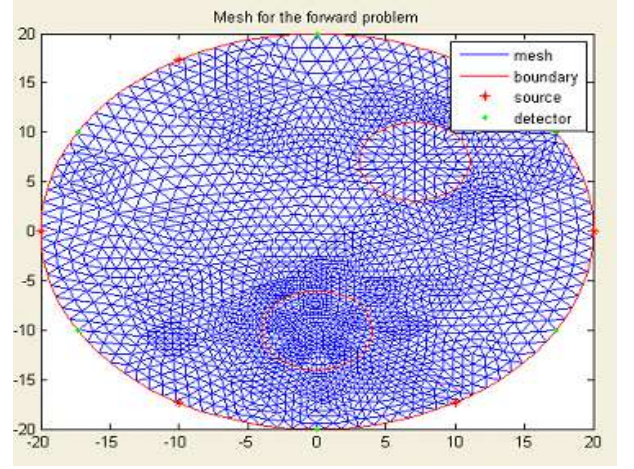


Fig 2. Mesh image of RTE for the forward problem

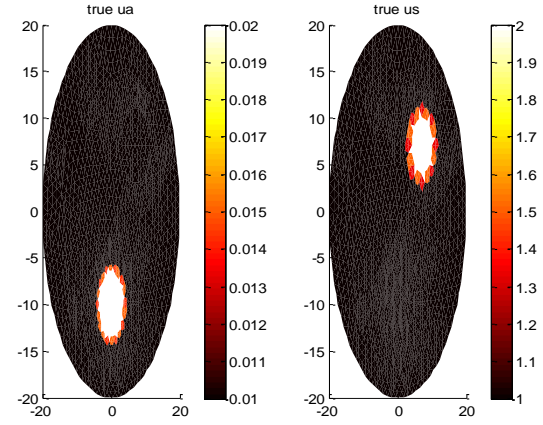


Fig 3. Real measurement of absorption and scattering coefficient for normal people.

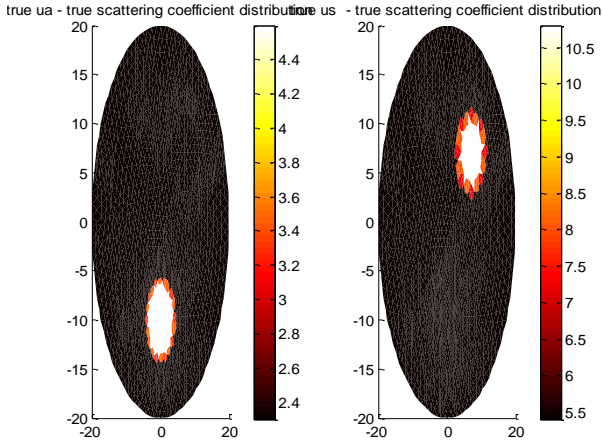


Fig 4. Real measurement of absorption and scattering coefficient for cancer affected people.

Figure 1 displays the angular discretization of the 2-D image and in this simulation which was obtained by placing the six detectors at the different angles on the tissue boundary. These phasor angles are at 0° or 360° , 45° , 135° , 180° , 225° and 315° respectively. Figure 2 display the 2-D circular mesh image for RTE. This type of circular mesh image is formed by the triangular element in our calculation with 1097 nodes and 9600 elements. In this method, the number of nodes in a forward problem for RTE is always greater than the number of nodes in the reconstructed mesh. It is used to improve our spatial resolution. Figure 3 shows the real measurement values of absorption and scattering coefficients for normal people ranges from 0.01 cm^{-1} to 1.0 cm^{-1} . Figure 4 shows the real measurement values of absorption and scattering coefficient for cancer affected people, it ranges from 2.4 cm^{-1} to 5.5 cm^{-1} respectively.

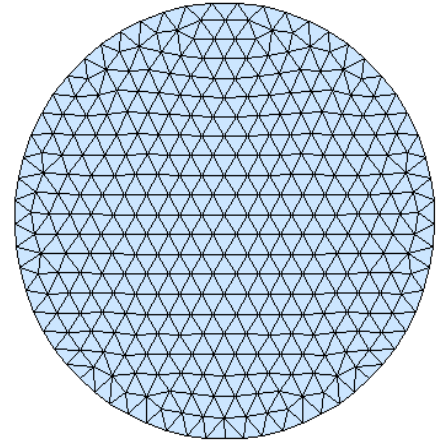


Fig 5. Mesh for FEM solution of the forward model

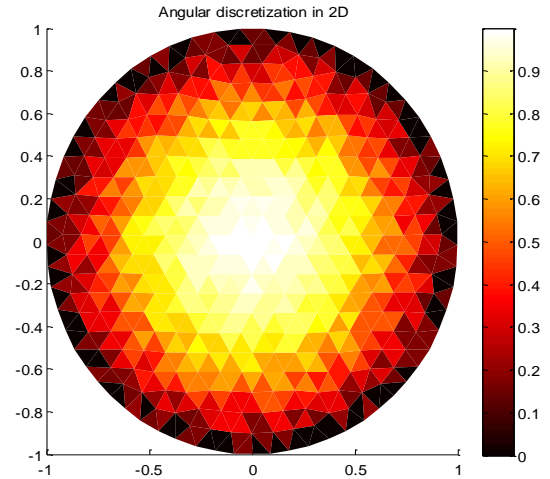


Fig 6. Angular discretization of the 2D image for FEM

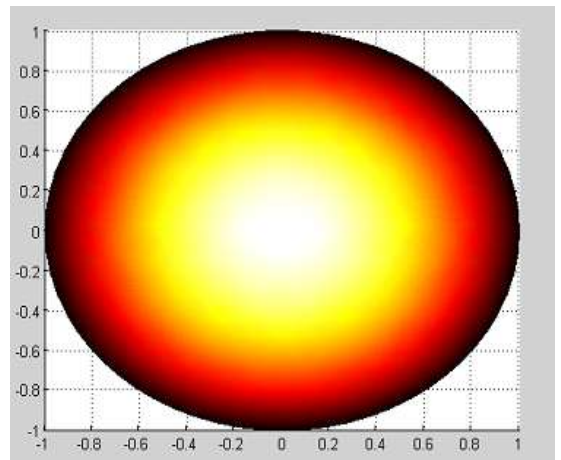


Fig 7. Magnitude of the optical properties using FEM

Figure 5 represents the two-dimensional circular mesh diagram formed by the triangular element for the FEM solution of the forward model. Figure 6 shows the angular discretization of the two-dimensional image for FEM using diffusion equation. Figure 7 represents the magnitude of the optical properties such as absorption and scattering coefficient for FEM.

TABLE II. ABSORPTION AND SCATTERING COEFFICIENT VALUES FOR NORMAL PEOPLE

S. No	Absorption coefficient (μ_a) cm^{-1}	Scattering coefficient (μ_s) cm^{-1}	Optical flux ϕ a.u.
1	0.01	1	1.18E-015
2	0.5	2.15	2.22E-016
3	0.8	2.7	1.97E-015
4	1.4	3.8	1.32E-015

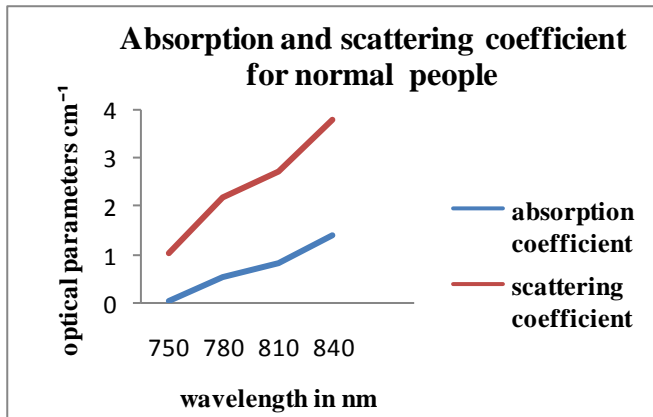


Fig 8. Absorption and scattering coefficient distribution for normal people

TABLE III. ABSORPTION AND SCATTERING COEFFICIENT VALUES FOR CANCER AFFECTED PEOPLE

S. No	Absorption coefficient (μ_a) cm^{-1}	Scattering coefficient (μ_s) cm^{-1}	Optical flux ϕ a.u.
1	2.4	5.7	3.25E-015
2	3.2	7	2.96E-015
3	4.7	10.3	2.74E-015
4	5.4	12.18	1.88E-015

Table II and Table III presents the absorption and scattering coefficient values along with optical flux reading for the normal people and cancer affected people as observed from the reconstructed image. We can distinguish the normal people and the affected people since normal people have absorption coefficient less than 2 cm^{-1} and scattering coefficient values less than 5 cm^{-1} . In the case of cancer affected people the absorption coefficient, exceeds 2 cm^{-1} as well as scattering coefficient exceeds 5 cm^{-1} . Figure 8 represents the optical parameters such as absorption coefficient in the range of 0.01 cm^{-1} to 1.4 cm^{-1} and the scattering coefficient in the range of 1 cm^{-1} to 3.8 cm^{-1} for normal people.

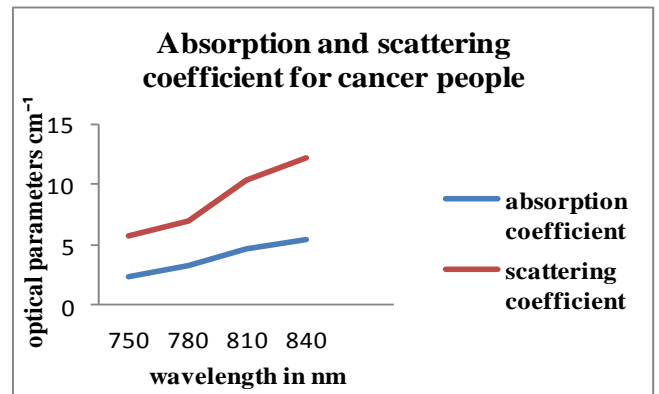


Fig 9. Absorption and scattering coefficient distribution for cancer people

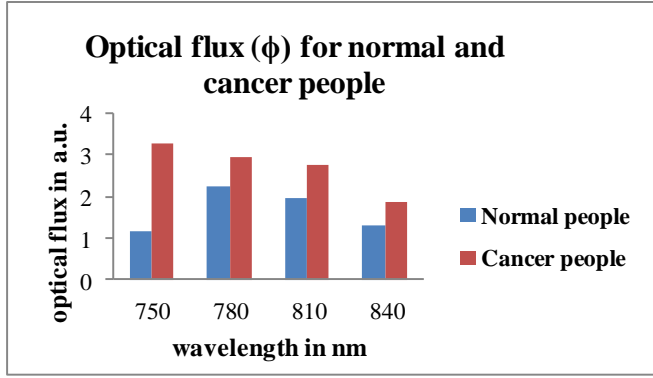


Fig 10. Optical flux (ϕ) value for normal and cancer affected people

Figure 9 represents the optical parameters such as absorption coefficient in the range of 2.4 cm^{-1} to 5.4 cm^{-1} and the scattering coefficient in the range of 5.7 cm^{-1} to 12.18 cm^{-1} in tumor patient. Figure 10 shows the optical flux value for normal and cancer affected people. The optical flux (ϕ) is energy per unit time that is radiated from a source over optical wavelengths in nm. The above graph shows the relationship between the optical flux and wavelengths in the range of 750 nm to 840 nm. The normal people will have lower flux value compared to cancer affected people. We can find the resolution of the reconstructed images by determining the signal to noise ratio (SNR) and CPU time for RTE and FEM. The SNR ratio is given as,

$$SNR = 10 \log_{10} \frac{\|signal\|_2}{\|noise\|_2} \quad (14)$$

TABLE IV. PERFORMANCE ANALYSIS OF FORWARD MODEL DESIGN

Parameters	Existing system	Methods	
		Radiative transfer equation (RTE)	Finite element method (FEM)
SNR	1.3321	1.9148	2.5039
CPU time	44.231	37.0645	35.7215

In, table IV parameters values are compared that is SNR and CPU time. The reconstructed image of an existing system provides low SNR ratio and higher CPU time about 44.231. The RTE gives 37.0645 CPU time and also provides higher SNR ratio compared than the existing system. Then, we compared to RTE, the FEM provides the higher SNR ratio and very less CPU time. Therefore, FEM is a most important method for improving system performance.

IV. CONCLUSION

By using algorithms in diffuse optical tomography (DOT) with NIR light ranging from 780 nm to 850 nm, we had performed the numerical simulation using the radiative transfer equation (RTE) and diffusion equation solved using finite element method (FEM) to find out the result of the soft tissue optical parameters that is absorption and scattering coefficient explicitly for normal and cancer affected people. The optical flux was also evaluated based on the forward model numerically. We had evaluated the performance of a system by varying the position of source and detectors, thereby calculating the geometrical symmetry of the tissue in a turbid medium. FEM is more efficient and flexible in implementing different boundary conditions. The parameter values such as SNR and CPU time are compared for both algorithms. FEM provides better performance and high rate of SNR value than RTE. By analyzing these values, Cancer affected tissue was predicted from the forward model, which in turn avoids the reconstruction of the image. It is proved that the FEM gives efficient results compared to RTE.

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